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WIMPs with mass ~100 GeV from a pair of massless and small-mass scalar-fields in interaction

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Abstract. – It is shown that a massless scalar-field can interact with a scalar-field bearing particles of mass *m* to produce localized particlelike concentrations of field energy, WIMP *solitons* with a mass *M* orders of magnitude greater than *m*. The characterizing signature features of such solitons are described, and their possible detection by ongoing observations is noted.

Tentative experimental evidence for WIMPs, Weak Interacting Massive Particles with a mass of the order 100 GeV, has been reported recently [1,2]. Conjectured to be the principal component of cold dark matter and hence about 84% of all matter, WIMPs are in striking contrast to the near-massless particles of previously considered scalar-field candidates for dark matter, fields which carry masses that range from $\sim 10^{-22}$ eV [3] to $\sim 10^{-3}$ eV [4]. Remarkably however, a pair of massless and small-mass scalar-fields in interaction can produce WIMPlike entities with a mass of the order 100 GeV, as shown below. Since observational evidence for self-interacting cold dark matter has been established for over a decade [5], this model for WIMPs is a viable alternative to a supersymmetry particle with WIMP properties.

Now after a century of particle-theory development, there has emerged a very large order-of-magnitude range in seemingly fundamental masses [6,7]. We have the hypothetical Planck mass $G^{-1/2} = 1.22 \times 10^{19}$ GeV [8], the (probably) massless photon ($m_\gamma \lesssim 2 \times 10^{-11}$ eV), the lightest mass-state neutrino ($m_{\nu_1} \lesssim 5 \times 10^{-3}$ eV), and finally the suspect-candidate dark matter particles, conjectured to be as large as ~ 100 GeV [1,2], as small as $\sim 10^{-3}$ eV [4], or perhaps even effectively massless at $\sim 10^{-22}$ eV [3]. Nestled in between and spanning five orders-of-magnitude from the weak-interaction gauge bosons with masses ~100 GeV down to the electron with

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$m_e = 0.511$ MeV, we have the experimentally established particles. In a future fundamental theory, the masses which enter must surely bear basic theoretical relationships to one another. Hence, considerable interest is attached to field-theoretic mechanisms that may account for order-of-magnitude differences in observable particle mass. The purpose of this communication is to describe such a mechanism, which may in fact apply to WIMPs.

Consider the effective (post-renormalization) Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{2} g^{\mu\nu} (\chi_{,\mu}\chi_{,\nu} + \phi_{,\mu}\phi_{,\nu}) - \frac{1}{2} m^2 \chi^2 + \beta\chi\phi^3 \tag{1}$$

in which χ is a real scalar-field bearing particles of mass m, ϕ is a massless real scalar-field, and β is a dimensionless coupling-constant. From (1) we obtain the field equations

$$\begin{aligned} &g^{\mu\upsilon}\chi_{,\mu\nu} - m^2\chi + \beta\phi^3 = 0 \\ &g^{\mu\upsilon}\phi_{,\mu\nu} + 3\beta\chi\phi^2 = 0 \end{aligned} \tag{2}$$

and the canonical energy density

$$T_{00} = \frac{1}{2}\Big((\chi_{,0})^2 + |\nabla\chi|^2 + (\phi_{,0})^2 + |\nabla\phi|^2 \Big) + \frac{1}{2} m^2\chi^2 - \beta\phi^3\chi \tag{3}$$

Let us suppose that Eqs. (2) admit a static solution with $\chi_{,0} = 0 = \phi_{,0}$ in an appropriate Lorentz frame, and let us seek a singularity-free solution with spherical symmetry about the point $x \equiv (x_1, x_2, x_3) = 0$. The first member of (2) can be recast as the integral equation

$$\chi(|x|) = \frac{\beta}{4\pi} \int \frac{(\exp - m|x-y|)}{|x-y|} \phi^3(|y|) d^3y \cong \beta m^{-2} \phi^3(|x|) \tag{4}$$

where the last member of (4) follows approximately if and only if

$$|\phi(|x|)/\phi(0)|^3 << 1 \qquad \text{for} \qquad |x| \gtrsim m^{-1} \tag{5}$$

i.e, if and only if $|\phi(|x|)/\phi(0)|$ declines from 1 to near 0 as a |x| increases from 0 to become greater than m^{-1}.

Then, with substitution of the last member of (4) into the second member of (2), we get the elliptic partial differential equation in the ϕ field exclusively,

$$\nabla^2\phi + 3\beta^2 m^{-2}\phi^5 = 0 \quad . \tag{6}$$

Remarkably, (6) admits the exact rigorous spherically-symmetric singularity-free *soliton* solution

$$\phi(|x|) = \pm\left(a|x|^2 + a^{-1}\beta^2 m^{-2}\right)^{-1/2} \tag{7}$$

in which *a* is a positive (dimensionless and disposable) constant of integration. We now verify *a posteriori* that for suitable values of *a* the solution (7) satisfies the approximation requirement (5):

$$\left|\phi(|x|)/\phi(0)\right|^3 = \left(a^2\beta^{-2}m^2|x|^2 + 1\right)^{-3/2} << 1 \quad \text{for} \quad |x| \gtrsim m^{-1} \tag{8}$$

Indeed, (8) holds if and only if

$$|\beta| << a \quad . \tag{9}$$

Since the approximate radial size of the solution (7) is given by $\hat{r} = a^{-1}|\beta|m^{-1}$, (9) states that radial size is sub-Compton, *i.e.*, small compared to m^{-1}. The total field energy mass of the soliton (7) follows from (3) and (4) as

$$\mathrm{M} \equiv \int \mathrm{T}_{00}\, d^3x = \pi^2 m/4|\beta| \tag{10}$$

with terms combining inside the integral, the subsequent integration being exact, and the constant of integration *a* scaling out. Hence, for $|\beta| << 1$ we have obtained solitons with a mass $M >> m$ (while conversely, for a superstrong coupling with $|\beta| >> 1$, we obtain $M << m$). It can be shown by long-established classical-field analysis that for all magnitudes of $|\beta|$ and a these solitons are anti-Coulombic, with like-sign solutions (7) attracting each other and unlike-sign solutions repelling each other [9]. The mass relation (10), the variable radial size feature of the solitons (with a disposable and $\hat{r} = \pi^2/4a\mathrm{M}$) and their anti-Coulombic character may provide an identifying signature for them. In particular, if dark matter WIMPs with $M \sim 100$ GeV interact with terrestrial matter to afford detection by the most advanced signature-determining *CDMS II* apparatus [10], and if they show the latter soliton signature features, then such WIMPs may derive from interaction between a pair of massless and small-mass dark matter scalar-fields , such as those conjectured on the basis of cosmological observations [3,4].

In summary, it has been shown that a massless scalar-field can interact with a scalar-field bearing particles of mass *m* to produce localized particlelike concentrations of field energy, *solitons* with a mass *M* orders of magnitude greater than *m*. The characterizing signature features of such solitons have been described, and such signature features may show up in the ongoing *CDMS II* observations [10]. If so, WIMPs with $M \sim 100$ GeV may be solitons that follow from the χ and ϕ fields, as derived rigorously here from the effective Lagrangian (1).

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