WIMPs with mass ~100 GeV from a pair of massless and small-mass scalar-fields in interaction

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Abstract. – It is shown that a massless scalar-field can interact with a scalar-field bearing particles of mass \( m \) to produce localized particlelike concentrations of field energy, WIMP solitons with a mass \( M \) orders of magnitude greater than \( m \). The characterizing signature features of such solitons are described, and their possible detection by ongoing observations is noted.

Tentative experimental evidence for WIMPs, Weak Interacting Massive Particles with a mass of the order 100 GeV, has been reported recently [1,2]. Conjectured to be the principal component of cold dark matter and hence about 84% of all matter, WIMPs are in striking contrast to the near-massless particles of previously considered scalar-field candidates for dark matter, fields which carry masses that range from ~10^{-22} eV [3] to ~10^{-3} eV [4]. Remarkably however, a pair of massless and small-mass scalar-fields in interaction can produce WIMP-like entities with a mass of the order 100 GeV, as shown below. Since observational evidence for self-interacting cold dark matter has been established for over a decade [5], this model for WIMPs is a viable alternative to a supersymmetry particle with WIMP properties.

Now after a century of particle-theory development, there has emerged a very large order-of-magnitude range in seemingly fundamental masses [6,7]. We have the hypothetical Planck mass \( G^{-\frac{1}{2}} = 1.22 \times 10^{19} \) GeV [8], the (probably) massless photon \( (m_{\gamma} \approx 2 \times 10^{-11} \) eV), the lightest mass-state neutrino \( (m_{\nu} \approx 5 \times 10^{-3} \) eV), and finally the suspect-candidate dark matter particles, conjectured to be as large as ~100 GeV [1,2], as small as ~10^{-3} eV [4], or perhaps even effectively massless at ~10^{-22} eV [3]. Nestled in between and spanning five orders-of-magnitude from the weak-interaction gauge bosons with masses ~100 GeV down to the electron with

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\( m_e = 0.511 \text{ MeV} \), we have the experimentally established particles. In a future fundamental theory, the masses which enter must surely bear basic theoretical relationships to one another. Hence, considerable interest is attached to field-theoretic mechanisms that may account for order-of-magnitude differences in observable particle mass. The purpose of this communication is to describe such a mechanism, which may in fact apply to WIMPs.

Consider the effective (post-renormalization) Lagrangian

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{2} \left( g_{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \beta \chi \phi \right) - \frac{1}{2} m^2 \chi^2 + \beta \chi \phi^3
\]

(1)

in which \( \chi \) is a real scalar-field bearing particles of mass \( m \), \( \phi \) is a massless real scalar-field, and \( \beta \) is a dimensionless coupling-constant. From (1) we obtain the field equations

\[
\partial_\mu \left( g_{\mu\nu} \partial_\nu \chi \right) = \beta \chi \phi \chi
\]

(2)

and the canonical energy density

\[
T_{00} = \frac{1}{2} \left( (\chi_0)^2 + |\nabla \chi|^2 + (\phi_0)^2 + |\nabla \phi|^2 \right) + \frac{1}{2} m^2 \chi^2 - \beta \chi \phi^3
\]

(3)

Let us suppose that Eqs. (2) admit a static solution with \( \chi,0 = \phi,0 = 0 \) in an appropriate Lorentz frame, and let us seek a singularity-free solution with spherical symmetry about the point \( x = (x_1, x_2, x_3) = 0 \). The first member of (2) can be recast as the integral equation

\[
\chi(x) = \frac{\beta}{4\pi} \int \frac{\text{exp} - m|x-y|}{|x-y|^3} \phi(y) \, dy \simeq \beta m^{-2} \phi(|x|)
\]

(4)

where the last member of (4) follows approximately if and only if

\[
|\phi(|x|)/\phi(0)|^3 << 1 \quad \text{for} \quad |x| \gtrsim m^{-1}
\]

(5)

i.e, if and only if \( |\phi(|x|)/\phi(0)| \) declines from 1 to near 0 as \( |x| \) increases from 0 to become greater than \( m^{-1} \).

Then, with substitution of the last member of (4) into the second member of (2), we get the elliptic partial differential equation in the \( \phi \) field exclusively,

\[
\nabla^2 \phi + 3 \beta^2 m^{-2} \phi^5 = 0
\]

(6)

Remarkably, (6) admits the exact rigorous spherically-symmetric singularity-free soliton solution
\[ \phi(x) = \pm \left( a|x|^2 + a^{-1} \beta^2 m^{-2} \right)^{-1/2} \]  

(7)

in which \( a \) is a positive (dimensionless and disposable) constant of integration. We now verify \( a \) \textit{a posteriori} that for suitable values of \( a \) the solution (7) satisfies the approximation requirement (5):

\[ \left| \phi(|x|)/\phi(0) \right|^3 = \left( \frac{2 \beta^2 m^{-2} |x|^2 + 1}{a^{-1} \beta^2 m^{-2}} \right)^{3/2} \ll 1 \quad \text{for} \quad |x| \gg m^{-1} \]  

(8)

Indeed, (8) holds if and only if

\[ |\beta| \ll a \]  

(9)

Since the approximate radial size of the solution (7) is given by \( \hat{r} = a^{-1} |\beta| m^{-1} \), (9) states that radial size is sub-Compton, \( i.e. \), small compared to \( m^{-1} \). The total field energy mass of the soliton (7) follows from (3) and (4) as

\[ M \equiv \int T_{00} \frac{3}{2} m_{f} \beta \]  

(10)

with terms combining inside the integral, the subsequent integration being exact, and the constant of integration \( a \) scaling out. Hence, for \( |\beta| \ll 1 \) we have obtained solitons with a mass \( M >> m \) (while conversely, for a superstrong coupling with \( |\beta| >> 1 \), we obtain \( M << m \)). It can be shown by long-established classical-field analysis that for all magnitudes of \( |\beta| \) and \( a \) these solitons are anti-Coulombic, with like-sign solutions (7) attracting each other and unlike-sign solutions repelling each other [9]. The mass relation (10), the variable radial size feature of the solitons (with \( a \) disposable and \( \hat{r} = \pi^2/4aM \)) and their anti-Coulombic character may provide an identifying signature for them. In particular, if dark matter WIMPs with \( M \sim 100 \text{ GeV} \) interact with terrestrial matter to afford detection by the most advanced signature-determining \textit{CDMS II} apparatus [10], and if they show the latter soliton signature features, then such WIMPs may derive from interaction between a pair of massless and small-mass dark matter scalar-fields, such as those conjectured on the basis of cosmological observations [3,4].

In summary, it has been shown that a massless scalar-field can interact with a scalar-field bearing particles of mass \( m \) to produce localized particlelike concentrations of field energy, \textit{solitons} with a mass \( M \) orders of magnitude greater than \( m \). The characterizing signature features of such solitons have been described, and such signature features may show up in the ongoing \textit{CDMS II} observations [10]. If so, WIMPs with \( M \sim 100 \text{ GeV} \) may be solitons that follow from the \( \chi \) and \( \phi \) fields, as derived rigorously here from the effective Lagrangian (1).
REFERENCES


