The Masses of Charged Leptons and Quarks from Superposition Self-interference of their Dirac Fields

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Abstract

Without Higgs field interaction, accurate pole mass values are obtained for the charged leptons and quarks from a Z_3-symmetric linear superposition self-interference of the Dirac fields in the effective free-field Lagrangian. The charged lepton and quark pole masses evidence the discrete Z_3 symmetry, the theoretical-experimental deviations $\delta m/m$ are $O(10^{-5})$ for all three charged leptons, and the quark pole masses are in very satisfactory overall agreement with the experimental data.

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1. Introduction.

While it is likely that a boson-interacting, SU(2) symmetry-breaking, Higgs will be discovered in forthcoming LHC experiments, it is quite uncertain whether the Higgs interacts with the fundamental fermion fields to produce their masses *via* Yukawa couplings. Indeed, the lepton and quark masses may arise alternatively by any one of several dynamical mechanisms. The purpose of the present communication is to show that accurate pole mass values appear for the charged leptons and quarks from a Z_3-symmetric linear superposition self-interference of the Dirac fields in the effective free-field Lagrangian, without Higgs interaction. This effective free-field Lagrangian with discrete Z_3 symmetry presumably arises through the dynamical mechanism in the primary Lagrangian, which effects a precise Z_3-symmetric superposition of the

Dirac fields. The basis for the present work derives in part from the results of Tarrach [1] and Kronfeld [2] on scale-independent pole mass in QCD and from the more recent empirical analysis and model theoretic studies by the present author [3]. Let us first briefly review the established status of scale-independent pole mass.

2. Pole mass in renormalizable field theory

The gauge independence of charged lepton mass was established in QED many years ago [4]. Subsequently, the IR finiteness and gauge independence of charged lepton mass was proven to all orders in QED [5]. Moreover, it has been known for many years that the pole mass is gauge-parameter independent in perturbative QCD, but it was not known whether the quark pole masses were IR finite until the early (somewhat conjectural) work of Tarrach [1], which inspired the rigorous proof of IR finiteness by Kronfeld [2]. The former author [1] made a "foolproof check" of the IR finiteness and gauge-parameter independence at the two-loop level, which was confirmed to be rigorous to all orders by the latter author [2]. Hence, since pole mass is also manifestly renormalization-scale independent, it is the unambiguously well-defined "mass" for quarks as well as for charged leptons, free of any scale-dependence. The practical problem of course is to determine the quark pole masses from baryonic experimental data. However, it has been shown that the pole masses for the u, d, s, c, b are in approximate correspondence with their respective "current masses" at the 2 GeV scale, while the pole mass of the t is the directly measured experimental value [1, 2], like pole masses of the charged leptons.

3. Effective free-field Lagrangian

With $\psi = \psi(x)$ a fermion Dirac field, let a phase-factor transformed physically-equivalent field be denoted by $\psi^{(\theta)} \equiv (\exp i\,\theta)\,\psi$, where the real constant θ is defined modulo 2π. Three specific values of θ are associated with the charged leptons and quarks, namely the θ values that follow from the tri-iterative Z_3 closure condition

$$\psi^{(3\theta)} = \psi^{(2/3)} \quad \Rightarrow \quad \theta = \theta_k = \frac{2\pi k}{3} + \frac{2}{9} \tag{1}$$

where the closure factor 2/3 is, by either curious coincidence or by physical connection, the average of the charge-number magnitudes ($|Q|$ = 1, 2/3, 1/3). In the second member of (1), the *generation index* k = 1, 2, 3 (modulo 3) for the 1st, 2nd and 3rd generations, respectively. The values of θ_k locate the vertices of an equilateral triangle on the U(1) gauge group circle, but this Z_3 symmetry is not a discrete subgroup of the U(1) continuous symmetry. Associated with the generation index k is the field Z_3 = {0, 1, 2} and the function mod_3 that maps an integer n into Z_3 : $mod_3(n)$ = 0, 1 or 2 (mod 3).

We postulate existence of a self-interference mechanism in the primary Lagrangian, a mechanism that splits a part of a fermion field into Aharanov-Bohm $\pm\theta_k$ phase-shifted components (due to the virtual electromagnetic vacuum) and then recombines them additively with their progenitor, like the action of a tri-refringent medium (with transmission intensities in the ratio 1 : ½ : ½) or a symmetrically tri-slitted screen (with aperture areas in the ratio 1 : ½ : ½). As the net result of the primary Lagrangian mechanism, a linear superposition of phase-shifted fields appears residually in the phenomenological (low-energy limit) effective free-field Lagrangian for each charged lepton and for each quark. The specific superposition for leptons and quarks is a linear combination of ψ_k , $\psi(\theta_k)$ and $\psi(-\theta_k)$, namely:

$$\psi'_k \equiv \psi_k + (1/\sqrt{2})(\psi(\theta_k) + \psi(-\theta_k)) = [1 + \sqrt{2}\cos\theta_k]\,\psi_k \quad , \qquad (2)$$

for each fermion with the generation index k = 1, 2, 3.

Now by letting $\psi_{|Q|,k}$ denote the Dirac fields for e, μ, τ with $|Q| = 1$, for u, c, t with $|Q| = 2/3$, and for d, s, b with $|Q| = 1/3$, the effective free-field Lagrangian is given by

$$\mathcal{L}_{\rm eff} = \sum_{k=1}^{3} \sum_{|Q|} \left(\overline{\psi}_{|Q|,k} \gamma^{\mu} \partial_{\mu} \psi_{|Q|,k} + \hat{m}\,|Q|\, (\exp \lambda_{|Q|,k})\, \overline{\psi}'_{|Q|,k}\, \psi'_{|Q|,k} \right) \qquad (3)$$

In (3), the contracted SU(3) color-indices on the quark fields $\psi_{2/3,k}$ and $\psi_{1/3,k}$ are tacit, and an empirical mass constant appears as

$$\hat{m} \equiv 313.85773 \text{ MeV} . \qquad (4)$$

Also in (3), there appears the Z_3-symmetric parameter $\lambda_{||Q|,k}$, which serves to effect RG invariance of the Tarrach-Kronfeld quark pole masses [1,2], as described in detailed by the latter authors. Empirically we find that $\lambda_{|Q|,k} = 3\,|B|\left(1 + 3\,|Q|\ mod_3\,(k \dotplus 3\,|Q|)\right)$, where $|B| = 3(|Q| - Q^2)/2$ is the magnitude of the baryon number for leptons or quarks. Hence,

$$\begin{aligned} &\lambda_{1,k} = 0 \\ &\lambda_{2/3,k} = 1 + 2\ mod_3\,(k+2) \\ &\lambda_{1/3,k} = 1 + mod_3\,(k+1) \end{aligned} \qquad (5)$$

in which the Z_3 symmetry produces an integer-valued quantization for the quark pole mass RG-invariance factor. Embodying self-interference of the Dirac fields as shown in (2), the effective free-field Lagrangian (3) reduces to

$$\mathcal{L}_{\rm eff} = \sum_{k=1}^{3} \sum_{|Q|} \overline{\psi}_{|Q|,k} (\gamma^{\mu} \partial_{\mu} + m_{|Q|,k})\, \psi_{|Q|,k} \qquad (6)$$

in which

$$m_{|Q|,k} = \hat{m}\,|Q|\ (\exp \lambda_{|Q|,k}\ )\,[1 + \sqrt{2}\ \cos \theta_k]^2 . \qquad (7)$$

Thus, the square-bracketed self-interference factor which appeared in (2) modulates the charged lepton and quark pole masses in (7).

4. Experimental correspondence

From (7) with $|Q| = 1$ for the charged leptons, we have (in units MeV)

$$
\begin{aligned}
m_{1,1} &= 0.51099651 = m_e\,(1 - 4.72\mathrm{x}10^{-6}) \\
m_{1,2} &= 105.65891 = m_\mu\,(1 + 5.10\mathrm{x}10^{-6}) \\
m_{1,3} &= 1776.9765 = m_\tau\,(1 - 7.61\mathrm{x}10^{-6})
\end{aligned}
\qquad (8)
$$

where m_e , m_μ , m_τ on the right sides of (8) are the precise experimental values [6]: m_e = 0.51099892 (1±7.8x10^{-8}), m_μ = 105.658369 (1±8.5x10^{-8}), and m_τ = 1776.99(1±1.58x10^{-4}) in MeV. A striking agreement with the experimental masses is evidenced by (8), with an O(10^{-5}) accuracy of correspondence. Hence, the charged lepton masses may be a manifestation of phase-structure and linear superposition self-interference in the effective free-field Lagrangian.

Likewise for the quark pole masses, (7), (5) and (8) yield

$$
m_{2/3,k} = \tfrac{2}{3}\left(\exp\left(1 + 2\,mod_3(\mathrm{k}+2)\right)\right) m_{1,k} = \begin{cases} 0.9260 & \text{for u} \quad (\mathrm{k}=1) \\ 1414.81 & \text{for c} \quad (\mathrm{k}=2) \\ 175{,}818 & \text{for t} \quad (\mathrm{k}=3) \end{cases}
$$

$$
m_{1/3,k} = \tfrac{1}{3}\left(\exp\left(1 + mod_3(\mathrm{k}+1)\right)\right) m_{1,k} = \begin{cases} 3.4212 & \text{for d} \quad (\mathrm{k}=1) \\ 95.737 & \text{for s} \quad (\mathrm{k}=2) \\ 4376.73 & \text{for b} \quad (\mathrm{k}=3) \end{cases}
\qquad (9)
$$

The latter theoretical values are in satisfactory agreement with the experimental data [6], with the Tarrach-Kronfeld pole masses for the u, d, s, c, b corresponding approximately to their respective “current” masses at the 2 GeV scale and the pole mass for the t given directly by its experimentally measured value [1, 2]. Most striking is the theoretical value for the top quark mass shown in (9), m_t = 175.818 GeV, which may in fact improve upon the directly measured experimental value [7], 175 GeV. It should be noted that the quark pole mass values in [9] differ somewhat from the values obtained in other physico-empirical models [3]; with improved experimental data, these differences in the theoretical quark pole mass values may serve to support the present or a former model.

Finally, it is interesting to note that (7) gives zero mass for the $|Q| = 0$ neutrinos, and the general formula for $\lambda_{|Q|,k}$ produces $\lambda_{0,k} = 0$. This requires an extension of (7) of order $(m_{0,k} / m_{1,k}) \sim O(10^{-10})$ to embrace the experimental neutrino masses [8]. A possible extension would require neutrinos to be Dirac and replace $|Q|$ with $\langle Q^2_{eff} \rangle^{1/2}$, where the latter *effective root-mean-square charge number* takes account of the virtual charge-distribution around a fermion due to electroweak closed-loop processes [9,10]. Then in general we have $\langle Q^2_{eff} \rangle^{1/2} = |Q| + O(10^{-10})$, and hence $\langle Q^2_{eff} \rangle^{1/2} \sim O(10^{-10})$ for $|Q| = 0$, with $\langle Q^2_{eff} \rangle^{1/2}$ modulating the so-called neutrino electroweak charge radii [9,10]. However, improved neutrino mass data and neutrino charge radii calculations are needed to test the $|Q| \rightarrow \langle Q^2_{eff} \rangle^{1/2}$ extension of (7).

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