EUROPHYSICS LETTERS

OFFPRINT

Vol. 62 • Number 4 • pp. 473–476

Semi-empirical operator for the self-interaction masses of finite-size leptons and quarks

* * *

G. Rosen

Published under the scientific responsability of the

EUROPEAN PHYSICAL SOCIETY

Incorporating

JOURNAL DE PHYSIQUE LETTRES • LETTERE AL NUOVO CIMENTO

EUROPHYSICS LETTERS 15 May 2003
Europhys. Lett., **62** (4), pp. 473–476 (2003)

Semi-empirical operator for the self-interaction masses of finite-size leptons and quarks

G. Rosen(*)
Department of Physics, Drexel University - Philadelphia, PA 19104, USA

(received 25 March 2002; accepted in final form 26 February 2003)

PACS. 12.15.Ff – Quark and lepton masses and mixing.

Abstract. – A semi-empirical operator is obtained for the self-interaction masses of finite-size leptons and quarks. Strong, electromagnetic and weak self-interaction energies are incorporated into the mass operator via the projection operators $P_1 = |B + Q|$ and $P_2 = |2B - Q|$ which act on the structural (*i.e.*, finite-size particle) quantum states for the fundamental fermions. The neutrino mass eigenvalues support the low-probability (LOW) solution for solar-neutrino oscillations, the charged lepton masses are given accurately to within $(\delta m/m) \sim 10^{-4}$, the top quark pole mass is predicted to be 174.241 GeV, and the other quark pole masses are uniformly consistent with experiment. Thus, the form of the mass operator may provide guidance to a field-theoretic extension of the standard model that embraces finite-size leptons and quarks.

Introduction. – While there is strong theoretical assurance that a Higgs boson exists to break the electroweak symmetry, its role as a possible provider of lepton-quark mass is conjectural and open to question. Instead, leptons and quarks may have a finite spatial size and associated finite self-interaction energies that essentially constitute their masses, as discussed recently by Goldhaber [1]. Fundamental fermions with a radial extension smaller than about 4×10^{-18} cm are indeed wholly admissible in light of the existing experimental resolution [2], and the masses of such finite-size entities may be a composite of strong, electromagnetic and weak interaction self-energies.

The present letter reports a semi-empirical operator for the self-energy generated masses of finite-size leptons and quarks, a self-interaction mass operator which acts as the rest-frame Hamiltonian for the structural entities themselves. Basic to the formulation are the empirical regularities and systematics in the charged lepton masses, in the quark pole masses, and in the neutrino mass eigenvalues [3–9]. The following *projection operator theorem* pertaining to the B and Q fermion quantum numbers serves as a prelude to the formulation of the self-interaction mass operator.

Projection operator theorem. – For the twelve lepton and quark structural states, the baryon number B, the surrogate for strong interaction, has the values

$$B = \begin{cases} 0 & \text{for } \nu_1, \nu_2, \nu_3, \mathrm{e}, \mu, \tau, \\ 1/3 & \text{for } \mathrm{d}, \mathrm{s}, \mathrm{b}, \mathrm{u}, \mathrm{c}, \mathrm{t}, \end{cases} \tag{1}$$

(*) Present address: 415 Charles Lane, Wynnewood, PA 19096, USA.

with $-B$ for the antiparticles, while the charge number Q, the surrogate for electromagnetic interaction, has the values

$$Q = \begin{cases} 0 & \text{for } \nu_1, \nu_2, \nu_3, \\ -1 & \text{for } \mathrm{e}, \mu, \tau, \\ -1/3 & \text{for } \mathrm{d}, \mathrm{s}, \mathrm{b}, \\ 2/3 & \text{for } \mathrm{u}, \mathrm{c}, \mathrm{t}, \end{cases} \tag{2}$$

with $-Q$ for the antiparticles. Let us introduce the associated quantities

$$P_1 \equiv |B + Q|, \qquad P_2 \equiv |2B - Q|, \tag{3}$$

where the absolute-value bars are understood in the eigenvalue sense for Hermitian (self-adjoint) operators: If $A = A^\dagger = \sum \lambda_k u_k u_k^\dagger$ with real eigenvalues $\{\lambda_k\}$ and normalized eigenvectors $\{u_k\}$, by definition $|A| \equiv \sum |\lambda_k| u_k u_k^\dagger$. Then the allowable lepton and quark structural state quantum numbers (1) and (2) follow if and only if P_1 and P_2 are projection operators with eigenvalues 0 and 1 on the structural quantum states,

$$P_1 = P_1^2 = \begin{cases} 0, \\ 1, \end{cases} \qquad P_2 = P_2^2 = \begin{cases} 0, \\ 1, \end{cases} \tag{4}$$

subject to the subsidiary condition for structural quantum states

$$B\, P_1 P_2 = 0. \tag{5}$$

Proof: That the conditions (4) and (5) are implied by (1) and (2) is seen directly by considering the four cases $(B, Q) = (0, 0)$, $(0, -1)$, $(\frac{1}{3}, -\frac{1}{3})$, $(\frac{1}{3}, \frac{2}{3})$ in (1) and (2). Conversely, that (4) subject to (5) implies (1) and (2) is shown by considering the four individual cases in (4). For example, $(P_1, P_2) = (0, 1)$ implies $Q = -B$ (from the first definition in (3)) which in turn requires $P_2 = 3|Q|$ and hence $Q = -B = \pm\frac{1}{3}$. Condition (5), automatically satisfied except for $(P_1, P_2) = (1, 1)$, precludes the unphysical (or at least so far unobserved) structural quantum state with $(B, Q) = (\frac{2}{3}, \frac{1}{3})$ and its antiparticle with $(B, Q) = (-\frac{2}{3}, -\frac{1}{3})$.

Self-interaction mass operator. – In light of the projection operator theorem, it is natural to view P_1 and P_2 defined by (3) as operators on the space of structural (*i.e.*, finite-size) lepton and quark states with the four sectors $(P_1, P_2) = (0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$ corresponding, respectively, to the fermions with $|Q| = 0$, $\frac{1}{3}$, $\frac{2}{3}$, 1. Commuting with P_1 and P_2, the self-interaction mass operator m gives the pole mass eigenvalues of unmixed eigenstates in the three sectors for which $P_1 P_2 = 0$ and the charged-lepton masses in the sector with $P_1 P_2 = 1$.

Since there are three leptons or quarks for each Q in (2), an additional generation-specifying operator must enter the structural model and commute with P_1 and P_2 in order to provide a complete set of eigenvalues that label the lepton-quark states. This generation-specifying operator is the *size index* S, interpreted physically below eqs. (9) and defined here by the operator equations

$$[P_1, S] = [P_2, S] = 0, \qquad (|S| - 1)(S + \sigma(3 + P_2)) = 0, \tag{6}$$

in which σ is the baryonic parity,

$$\sigma \equiv (1 - 2P_1)(1 - 2P_2) = \begin{cases} +1 & \text{for the lepton states } \nu_1, \nu_2, \nu_3, \mathrm{e}, \mu, \tau, \\ -1 & \text{for the quark states } \mathrm{d}, \mathrm{s}, \mathrm{b}, \mathrm{u}, \mathrm{c}, \mathrm{t}. \end{cases} \tag{7}$$

TABLE I – *Quantum numbers and self-interaction masses (obtained from* (8) *with* (9)*) for the finite-size lepton and quark structural states. Units are* MeV *except for the neutrinos, where* eV *magnitudes are indicated.*

	ν_1	ν_2	ν_3	e	μ	τ
Q	0	0	0	-1	-1	-1
P_1	0	0	0	1	1	1
P_2	0	0	0	1	1	1
S	-3	-1	1	-4	-1	1
m	3.09×10^{-7} eV	2.04×10^{-4} eV	5.75×10^{-2} eV	0.511131	105.683	1776.53
	d	s	b	u	c	t
Q	$-1/3$	$-1/3$	$-1/3$	$2/3$	$2/3$	$2/3$
P_1	0	0	0	1	1	1
P_2	1	1	1	0	0	0
S	-1	1	4	-1	1	3
m	11.743	197.39	4534.8	5.0916	1438.8	174241

With P_1 and P_2 replaced by their respective eigenvalues 0 or 1, the final (secular equation) member of (6) gives the eigenvalues of S as 1, -1, and $-\sigma(3 + P_2)$. Table I shows the eigenvalue quantum numbers Q, P_1, P_2, and S for the twelve leptons and quarks. Fermion symbol assignments are indicated above the columns by the mass values obtained from (8) below, which are also displayed in the table.

The structural-state operators Q, P_1, P_2, and S have been employed in an *ansatz* for the self-interaction mass operator m. Statistical analysis [10,11] of the lepton-quark mass data [8] suggests the form of the *ansatz*, and it is subsequently fixed empirically as

$$m = \overline{m}(2|S| + 1)^{-1}\left[\left(Q^2 + \varepsilon\right)(41/10)^S\right]^{2-P_2}. \tag{8}$$

In (8) there appear the characteristic *electromagnetic-strong interaction mass* $\overline{m}$ and the *weak interaction mass-fraction* ε, constant parameters with the empirical values

$$\overline{m} = 1299.90\,\text{MeV}, \qquad \varepsilon \cong 2.81 \times 10^{-6}. \tag{9}$$

Also appearing in (8) is the integer ratio (41/10), a scaling factor already encountered in a more limited context by Sirlin [4]. The size index S, appearing in the prefactor $(2|S| + 1)^{-1}$ and as a scaling exponent in (8), relates the size-associated enhancement or diminution of the self-energies by the positive or negative integer eigenvalues of S; the base factor for this size-associated enhancement or diminution is (41/10) for both the electromagnetic-strong and the weak self-interaction energies. Presumably the quantum number S and the base factor (41/10) stem from the self-similar geometrical size change of the leptons and quarks from generation to generation, which effects enhancement or diminution of the self-interaction energies in the manner discussed qualitatively by Goldhaber [1]. Finally, the order index $(2 - P_2)$ appears as the overall exponent on the square bracket in (8), with the self-interaction energies featuring a dominant electromagnetic-strong term proportional to Q^2 for $P_2 = 1$ structural states (e, μ, τ, d, s, b) and Q^4 for $P_2 = 0$ structural states (u, c, t with $Q = 0$ for the ν's).

Self-interaction mass values. – The theoretical mass values obtained from (8) with (9) are displayed in table I. All of these self-interaction mass values are in very satisfactory agreement with the experimental lepton masses and quark pole mass values [8, 9]. In particular, the experimental fractional deviations for the charged leptons are all of the order 10^{-4}:

$$\delta m_{\mathrm{e}}/m_{\mathrm{e}} = 2.6 \times 10^{-4}, \qquad \delta m_{\mu}/m_{\mu} = 2.3 \times 10^{-4}, \qquad \delta m_{\tau}/m_{\tau} = -2.6 \times 10^{-4}. \tag{10}$$

Of special contemporary experimental interest are the neutrino mass eigenvalues $m_1 = 3.09 \times 10^{-7}$ eV, $m_2 = 2.04 \times 10^{-4}$ eV, and $m_3 = 5.75 \times 10^{-2}$ eV, which are consistent with $m_3^2 - m_2^2 \cong m_3^2 - m_1^2 \cong 3.3 \times 10^{-3}\,(\mathrm{eV})^2$ for atmospheric-neutrino oscillations and with $m_2^2 - m_1^2 \cong 4.2 \times 10^{-8}\,(\mathrm{eV})^2$ for the low-probability (LOW) solution to the solar-neutrino oscillation data [9]. The quark pole masses given by (8) are likewise uniformly consistent with the experimental data [8], and the predicted value for the top quark pole mass $m_{\mathrm{t}} = 174.241$ GeV may in fact prove to be accurate to four (or more) significant figures.

In conclusion, the self-interaction mass operator (8) works in a striking manner to yield accurate mass values for the leptons and quarks. Thus, the operator (8) is an admissible rest-frame Hamiltonian for finite-size fundamental fermions on the probable scale $\sim 10^{-18}$ cm. The form of (8) may provide guidance to a field-theoretic extension of the standard model that embraces finite-size leptons and quarks.

REFERENCES

[1] Goldhaber M., *Proc. Natl. Acad. Sci.*, **99** (2002) 33.
[2] CDF Collaboration, *Phys. Rev. Lett.*, **87** (2001) 231803.
[3] Koide Y., *Phys. Rev. D.*, **49** (1994) 2638.
[4] Sirlin A., *Comm. Nucl. Part. Phys.*, **21** (1994) 227.
[5] Rosen G., *Mod. Phys. Lett. A*, **11** (1996) 1687.
[6] Palladino B. E. and Ferriera P. L., *Nuovo Cimento A*, **110** (1997) 303.
[7] Rosen G., *J. Difference Eq. Appl.*, **8** (2002) 25.
[8] Groom D. E. *et al.*, *Eur. Phys. J. C*, **15** (2000) 1; `http://pdg.lbl.gov/`.
[9] Linder M. and Winter W., *Phys. Rev. D.*, **66** (2002) 057303; Totsuka Y., `http://neutrinohouches.in2p3.fr/`; Heeger K., `http://neutrinohouches.in2p3.fr/`.
[10] Tarantola A., *Inverse Problem Theory - Methods for Data Fitting and Model Parameter Estimation* (Elsevier, Amsterdam) 1987.
[11] Vapnik V. N., *Statistical Learning Theory* (Wiley, New York) 1998.