QCD, quantum gravity, and the size of the proton

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Abstract. – Quantum chromodynamics and quantum gravity are essentially intertwined, according to brane theory. As a consequence, quantum gravitational vacuum fluctuations may have increased the proton's volume cosmologically at the rate $G\hbar/4\pi c^2$, thereby producing the proton's root-mean-square-radius $\hat{r}_p = 0.863(\pm.008)$ *fm* at the present cosmic time $\hat{t} = 13.7(\pm 0.4)$ *Gyr*.

With refined lattice calculations, nonperturbative QCD now provides a very satisfactory description of the quark-gluon proton structure [1,2]. However, on length scales twenty orders of magnitude greater than the reduced Planck-Weyl length $\ell_G \equiv \sqrt{G\hbar/4\pi c^3} = 0.45593\ x10^{-33}\,cm$ [3], brane theory [4−7] suggests that gravitational forces are likely to be many orders of magnitude stronger than given classically by Einstein's theory. The seven internal curled spatial dimensions lead to the formation of *moduli fields*, which generate strong *gluon moduli forces* on the ~ 1 *fm* scale, without showing any detectible modification to classical relativity on scales greater than $\sim 10^{-3}$ *cm*, [8]. If QCD were instead unaltered by quantum gravity, the universal character of the proton mass and the constituent quark masses would set the proton's size as an associated universal constant, *viz.*, the value established experimentally for the proton's root-mean-square-charge radius $\hat{r}_p = 0.86(\pm.02)$ *fm*, [9−13]. However, with the inclusion of quantum gravity, the size of the proton is quite likely to be a cosmological variable. Moreover, since the proton is the basic mass-constituent in the visible Universe, its size variation may be the primary observable hallmark of quantum gravity.

Can astronomical observations and cosmological theory throw light on whether quantum gravity effects the size of the proton? Spectroscopic astronomical measurements have confirmed the overall

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consistency of cosmological theory, with the fundamental constants c, $\hbar$, G, and the elementary particle masses and electric charges the same in all observed galaxies, and hence the same throughout the Universe's $\hat{t} = 13.7(\pm 0.4)\ Gyr$ lifetime. On the other hand, it is not possible to infer from spectroscopic astronomical measurements whether the proton's root-mean-square charge radius r_p is a universal constant, other than to assert that $r_p \lesssim 1\ fm$ for protons in all galaxies that radiate the hydrogen frequencies. Hence, it is consistent with observation that the proton size has been a cosmological variable influenced by quantum gravitational vacuum fluctuations over the course of the Universe's history.

What is the simplest law that might govern changes in the proton's volume $V_p = 4\pi r_p^3/3$ as a function of cosmic time t, due to quantum gravitational vacuum fluctuations characterized by ℓ_G? Surface growth and dimensional considerations immediately lead to the simple rate equation

$$dV_p/dt = c\,\ell_G^2 = G\hbar/4\pi c^2 \tag{1}$$

Integrating the latter rate equation subject to the initial condition $V_p = 0$ [or perhaps $V_p \sim 0(\ell_G^3)$] at $t = 0$, we have

$$V_p = G\hbar t/4\pi c^2 \tag{2}$$

and hence with $t = \hat{t}$, we find the present value of the proton's radius

$$\hat{r}_p = \left(3\hat{V}_p/4\pi\right)^{1/3} = \left(3G\hbar/16\pi^2 c^3\right)^{1/3}\left(c\hat{t}\right)^{1/3}$$
$$= 0.863(\pm .008)\ fm \tag{3}$$

where the uncertainty shown in (3) is due entirely to the uncertainty in $\hat{t}$, muted by the one-third power.

The final value in (3) falls between the CODATA experimental value [12]

$$\hat{r}_p = 0.8768\ \ fm \tag{4}$$

(which is now reinforced somewhat by [13]), and the recent muonic hydrogen experimental value [9]

$$\hat{r}_p = 0.8418\ \ fm \tag{5}$$

Support for the quantum gravity volume-growth-rate equation (1) would come from the eventual convergence of the experimental value for $\hat{r}_p$ to the theoretical value (3). If indeed improved measurements confirm the theoretical value (3), then we would have evidence for the intertwining of the nonlinear theories of QCD and quantum gravity, with the latter as protagonist in determining the size of the proton.

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