On the Classical to Quantum Correspondence

Although the article by Effros [1] is specifically targeted for *young (naïve?) mathematicians*, this old mathematical/theoretical physicist* was surprised to see no mention or reference to Dirac's co-discovery and subsequent refined development [2] of the Poisson bracket to commutator bracket correspondence for quantization. Indeed Born, who is credited by Effros as the sole originator of the quantization correspondence, states that [3] :

> "These commutation laws (Born and Jordan, 1925) take the place of the quantum conditions in Bohr's theory....It may be mentioned in conclusion that this fundamental idea underlying Heisenberg's work has been worked out by Dirac (1925) in a very original way."

Young mathematicians might also enjoy the fact exploited in the 1960's (see for example [4]) that the Poisson and commutator brackets are both *Lie product* binary operations with the properties:

$$[A + B, C] = [A, C] + [B, C] \qquad \text{(linearity)}$$

$$[A, B] \equiv -[B, A] \qquad \text{(antisymmetry)}$$

$$[[A, B], C] + [[B, C], A] + [[C, A], B] \equiv 0 \qquad \text{(integrability)}$$

In addition, both the Poisson and commutator brackets have the property

$$[AB, C] = A[B, C] + [A, C]B$$

with the direct product of the algebraic elements defined appropriately in either case. Hence, classical mechanics features the Lie product according to Poisson, while quantum mechanics has the Lie product represented by a commutator of linear operators. This takes some of the mystery out of the correspondence, at least for those who share a predilection for algebraic aesthetics.

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[1] E. G. Effros, Matrix revolutions: an introduction to quantum variables for young mathematicians, *Mathematical Intelligence* 26 (2004), 53-59.
[2] P. A. M. Dirac, *Quantum Mechanics*, Clarendon Press (1930), particularly chapter 4.
[3] M. Born, *Atomic Physics*, Hafner Publishing Co (1957), p. 130.
[4] G. Rosen, *Formulations of Quantum and Classical Dynamical Theory*, Academic Press (1964).

*A *mathematical physicist* is one who doesn't have the skills to do *real* mathematics; a *theoretical physicist* is one who doesn't have the skills to do *real* experiments.