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Mass Empirics of Leptons and Quarks: the Primary Clue to a Structural Theory for the Fundamental Fermions

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ABSTRACT

As a long-standing deficiency, the self-interaction mass divergences are side-stepped but not resolved in a physical manner by the renormalization procedure in quantum electrodynamics and its multi-gauge-invariant Standard Model (SM) extension for the electroweak and strong interactions. While the existing theory does not yield finite physical predictions for self-interaction mass, experiments have revealed scale-related regularities in the lepton and quark mass spectra, suggesting that these masses embody electromagnetic, weak and strong self-interaction energies. Moreover, these scale-related regularities are expressed compactly by a simple semi-empirical operator for lepton and quark pole mass. The probable hallmark of fundamental fermions with finite self-interaction masses and finite spatial extension, the semi-empirical mass operator may be a primary clue to an underlying theory which will yield the mass operator directly while embracing SM in the formal point-particle approximation $\ell \to 0$, where ℓ is the fundamental length constant associated with fermion size. The actual physical value of ℓ is conjectured to be approximately 0.57×10^{-18} cm on the basis of empirical constants that appear in the mass operator.

INTRODUCTION

From its original inception, point-particle Quantum Field Theory (QFT) brought in singularities on the lightcone $(x' - x'')^2 = 0$ in the commutation and anticommutation relations for boson and fermion field operators at spacetime points x' and x'' [*1–5*]. In combination with coupling-term perturbation theory, these singularities on the lightcone generated unphysical infinites for the self-interaction masses of the electron and the other fundamental fermions. Nevertheless, all statistical scattering process and particle-transmutation predictions of the principal QFT's, quantum electrodynamics and its Standard Model (SM) extension for the electroweak and strong interactions, have proved to be remarkably accurate. Moreover, at an early date [*6*] it was realized that the small energy shifts produced in the electron's self-interaction mass by external electric and/or magnetic fields could be extracted by a formal iterative *renormalization* subtraction technique and compared with experiment; here again, the theoretical predictions were remarkably accurate. This unnatural situation of finite physical increments residing within blatantly infinite quantities persisted through the 20$^{\text{th}}$-century, and indeed the admissibility of renormalization was a touchstone in the formulation of the multiple-gauge-invariant SM. But many mathematical physicists, including Dirac [*7*], have viewed the renormalization subtract procedure to be basically provisional:

> "It seems to be quite impossible to put this theory on a mathematically sound basis. At one time physical theory was all built on mathematics that was inherently sound. I do not say that physicists always use sound mathematics; they often use unsound steps in their calculations. But previously when they did so it was simply because of, one might say, laziness. They wanted to get results as quickly as possible without doing unnecessary work. It was always possible for the pure mathematician to come along and make the theory sound by bringing in further steps, and perhaps by introducing quite a lot of cumbersome notation and other things that are desirable from a mathematical point of view in order to get everything expressed rigorously but do not contribute to the physical ideas. The earlier mathematics could always be made sound in that way, but in the renormalization theory we have a theory that has defied all attempts of the mathematician to make it sound. I am inclined to suspect that the renormalization theory is something that will not survive in the future,…."

Forty years have passed since the above passage was written, but it still describes the essence of the present situation. The singularities on the lightcone for point-particle fermions give rise to infinite unphysical self-interaction energies. Nevertheless, many authors* [*e.g.*, *8–19* and works cited therein] continue to hope that a suitable renormalizable extension of SM with point-particle fermions will yield a complete theory for leptons and quarks, with the original and long-standing QFT lightcone singularity deficiency swept under the rug by the renormalization procedure. Opportunely, another seemingly more practical route to the underlying physical theory may be available.

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* Other authors currently pursue quasi-nonphysical candidates for an SM extension, such as higher-dimensional (string *et al.*) QFT's which may feature characteristic length-scales of the order 10^{-33} cm, fifteen orders of magnitude smaller than current or foreseeable experimental resolution. Such extrapolations of established theory to higher dimensions or by fifteen orders of magnitude appear to be unmotivated by any physico-empirical evidence.

Empirical-Analysis and Formula-Discovery Program for a Lepton-Quark Structural Theory

The structure of finite-size leptons and quarks may be described by an underlying theory which yields accurate values for the lepton and quark self-interaction pole masses. The underlying theory would embrace SM in the formal point-particle approximation with $\ell \to 0$, where ℓ is the fundamental length constant that is associated with fermion size, conjectured here to be of the order 10^{-18} cm. It is helpful to recall the empirical-analysis and formula-discovery program on atomic spectra, the work of Ritz, Balmer and others, which led to the quantum theory for atomic structure 80 years ago. Analogously, if indeed a progenitor quantum theory generates the structural properties of leptons and quarks, then the immediate clue to such a theory is to be found in the mass empirics of the fundamental fermions. Fortunately, nature can be expected to manifest aesthetic mathematical simplicity in the equations of such a basic theory. Hence, a structural theory for leptons and quarks may be obtainable by studying the regularities and systematics in their principal empirical structural quantities, their pole masses m, quantities manifestly dependent on the charge numbers Q and baryon numbers B for the three generations of fermions. Recent progress toward the formulation of such an underlying theory for leptons and quarks is reviewed in the following.

The Physical Picture

Fundamental fermions with a radial extension smaller than about 4×10^{-18} cm are wholly admissible in light of the existing experimental resolution [*20*]. Thus leptons and quarks may have a finite spatial size and associated finite self-interaction energies that contribute either positively or in a negative binding sense to their masses [*21* and works cited therein]. The unmixed neutrino mass eigenstates ν_1, ν_2, ν_3 may derive their very small masses from their weak self-interaction binding, for they do not participate in electromagnetic or strong interactions. The charged leptons e, μ, τ may derive their masses in part from their electromagnetic self-interaction energies, along with an admixture of weak self-interaction binding energy, for the charged leptons do not participate in strong interactions. Finally, the unmixed quarks u, d, s, c, b, t may derive their eigenstate pole masses in part from their strong self-interaction energies, along with some electromagnetic and weak self-interaction energies.

The three generations of self-interaction modified masses may all stem from a primary *bare fermion mass* $\overline{m}$, a constant of the order ~ 10^3 MeV, with $\overline{m}$ appearing in the QFT approximation to the underlying theory as a common bare mass for all 12 Dirac fermions (with 4 internal-space components for each lepton and 3x4 internal-space components for each tricolored quark). Self-interaction energies in combination with self-similar geometrical size changes either suppress or elevate $\overline{m}$ to the observed fermion pole masses. In particular, the observed neutrino masses emerge as $\varepsilon^2\overline{m}$ modulo geometrical size-change factors, where ε is the *weak interaction suppression factor*, a dimensionless constant of the order $\sim 10^{-5}$.

Empirical analysis of the lepton and quark mass values reveal regularities and systematics that admit formulaic expression [*22–28*]. This empirical-analysis and formula-discovery program suggests that a *self-interaction mass operator* is the central physical object in the underlying theory. As the rest-frame Hamiltonian for the structure of finite-size leptons and quarks, the self-interaction mass operator has eigenvalues that give the masses of the charged leptons and the pole masses of the unmixed mass-eigenstate neutrinos and quarks. The following *projection operator theorem* [*29*] serves as a prelude to the formulation of the self-interaction mass operator.

PROJECTION OPERATOR THEOREM

For the twelve lepton and quark structural states, the baryon number B, the surrogate for strong interaction, has the values

$$B = \begin{cases} 0 & \text{for } \nu_1, \nu_2, \nu_3, e, \mu, \tau \\ 1/3 & \text{for } d, s, b, u, c, t \end{cases} \tag{1}$$

with –B for the antiparticles, while the charge number Q, the surrogate for electromagnetic interaction, has the values

$$Q = \begin{cases} 0 & \text{for } \nu_1, \nu_2, \nu_3 \\ -1 & \text{for } e, \mu, \tau \\ -1/3 & \text{for } d, s, b \\ 2/3 & \text{for } u, c, t \end{cases} \qquad (2)$$

with – Q for the antiparticles. Let us introduce the associated quantities

$$P_1 \equiv |B + Q| \qquad\qquad P_2 \equiv |2B - Q| \qquad (3)$$

where the absolute value bars are understood in the eigenvalue sense for Hermitian (self-adjoint) operators: If $A = A^\dagger = \Sigma\, \lambda_k u_k u^\dagger_k$ with real eigenvalues $\{ \lambda_k \}$ and normalized eigenvectors $\{ u_k \}$, by definition $|A| \equiv \Sigma |\lambda_k|\, u_k u^\dagger_k$. Then the allowable lepton and quark structural state quantum numbers (1) and (2) follow if and only if P_1 and P_2 are projection operators with eigenvalues 0 and 1 on the structural quantum states

$$P_1 = P_1^2 = \begin{matrix} 0 \\ 1 \end{matrix} \qquad\qquad P_2 = P_2^2 = \begin{matrix} 0 \\ 1 \end{matrix} \qquad (4)$$

subject to the subsidiary condition for structural quantum states

$$B\, P_1\, P_2 = 0 \qquad (5)$$

Proof: That the conditions (4) and (5) are implied by (1) and (2) is seen directly by considering the four cases (B, Q) = (0, 0), (0, –1), ($\frac{1}{3}$,– $\frac{1}{3}$), ($\frac{1}{3}$,$\frac{2}{3}$) in (1) and (2). Conversely, that (4) subject to (5) implies (1) and (2) is shown by considering the four individual cases in (4). For example, (P_1, P_2) = (0, 1) implies Q = – B [from the first definition in (3)] which in turn requires $P_2 = 3\,|Q|$ and hence Q = – B = ± $\frac{1}{3}$. Condition (5), automatically satisfied except for (P_1, P_2) = (1, 1), precludes the unphysical (or at least so far unobserved) structural quantum state with (B, Q) = ($\frac{2}{3}$, $\frac{1}{3}$) and its antiparticle with (B, Q) = (– $\frac{2}{3}$, – $\frac{1}{3}$).

Self-Interaction Mass Operator

In light of the projection operator theorem, it is natural to view P_1 and P_2 defined by (3) as operators on the space of structural (*i.e.*, finite-size) lepton and quark states with the four sectors $(P_1, P_2) = (0, 0), (0, 1), (1, 0), (1, 1)$ corresponding respectively to the fermions with $|Q| = 0, \frac{1}{3}, \frac{2}{3}, 1$. Commuting with P_1 and P_2, the self-interaction mass operator m gives the pole mass eigenvalues of unmixed eigenstates in the three sectors for which $P_1 P_2 = 0$ and the charged lepton masses in the sector with $P_1 P_2 = 1$.

Since there are three leptons or quarks for each Q in (2), an additional generation-specifying operator must enter the structural model and commute with P_1 and P_2 in order to provide a complete set of eigenvalues that label the lepton-quark states. This generation-specifying operator is the *size index* S, interpreted physically below Eqs. (10) and defined here by the operator equations [*29*]

$$[P_1, S] = [P_2, S] = 0 \qquad\qquad (|S|-1)\left(S+4\sigma+P\right) = 0 \tag{6}$$

in which σ is the baryonic parity,

$$\sigma \equiv (-1)^{3B} \equiv (1-2P_1)(1-2P_2) = \begin{cases} +1 \text{ for the lepton states } \nu_1, \nu_2, \nu_3, e, \mu, \tau \\ -1 \text{ for the quark states d, s, b, u, c, t} \end{cases} \tag{7}$$

and

$$P \equiv 3B(Q+B) \equiv P_1(1-P_2) = \begin{cases} 1 \quad \text{for } |Q| = \frac{2}{3} \\ 0 \quad \text{otherwise} \end{cases} \tag{8}$$

is the composite projection operator on the $|Q| = \frac{2}{3}$ sector. With P_1 and P_2 replaced by their respective eigenvalues 0 or 1, the final (secular equation) member of (6) gives the eigenvalues of

S as 1, –1, and $-(4\sigma+P)$. The Table shows the eigenvalue quantum numbers Q, P_1, P_2, and S for the twelve leptons and quarks. Fermion symbol assignments are indicated above the columns by the mass values obtained from (9) below, which are also displayed in the Table.

The structural-state operators Q, P_1, P_2, and S have been employed in an *ansatz* for the self-interaction mass operator m. Statistical analysis [*30*, *31*] of the lepton-quark mass data [*32*–*36*] suggests the form of the *ansatz*, and it is subsequently fixed empirically as [*29*]

$$m = \overline{m}(2|S|+1)^{-1}\left[(Q^2+\varepsilon^2)(41/10)^S\right]^{1+P} \tag{9}$$

In (9) there appear the characteristic *bare fermion mass* $\overline{m}$ and the *weak interaction suppression factor* ε , constant parameters with the empirical values

$$\overline{m} = 1299.90 \text{ MeV} \qquad\qquad \varepsilon \cong 6.0\times10^{-6} \tag{10}$$

Also appearing in (9) is the integer ratio (41/10), a scaling factor for mass already encountered in the more limited charged-lepton context [*23*]. The size index S, appearing in the prefactor $(2|S|+1)^{-1}$ and as a scaling exponent in (9), relates the size-associated enhancement or diminution of the self-energies by the positive or negative integer eigenvalues of S; the base factor for this size-associated enhancement or diminution is (41/10) for the electromagnetic, strong and the weak self-interaction energies. Presumably the quantum number S and the base factor (41/10) stem from the self-similar geometrical size change of the leptons and quarks from generation to generation, which effects enhancement or diminution of the self-interaction energies in a manner already proposed and discussed qualitatively [*21*]. Finally, the order index $(1+P)$ appears as the overall exponent on the square-bracket in (9), with the self-interaction energies featuring an electroweak term proportional to $(Q^2+\varepsilon^2)$ for the P = 0 structural states (ν_1, ν_2, ν_3, e, μ, τ, d, s, b) and $(Q^2+\varepsilon^2)^2 \cong 16/81$ for the P = 1 structural states (u, c, t) with $|Q| = \frac{2}{3}$.

SELF-INTERACTION MASS VALUES

The theoretical mass values obtained from (9) with (10) are displayed in the Table. All of these self-interaction mass values are in very satisfactory agreement with the experimental lepton masses and quark pole mass values [*32–36*]. In particular, the experimental fractional deviations for the charged leptons are all of the order 10^{-4}:

$$\delta m_e/m_e = 2.6 \times 10^{-4} \qquad \delta m_\mu/m_\mu = 2.3 \times 10^{-4} \qquad \delta m_\tau/m_\tau = -2.6 \times 10^{-4} \tag{11}$$

Of special contemporary experimental interest are the neutrino mass eigenvalues $m_1 = 1.8\text{x}10^{-5}$ eV, $m_2 = 3.8\text{x}10^{-3}$ eV, and $m_3 = 6.4\text{x}10^{-2}$ eV, which are consistent with $m_3^2 - m_2^2 \cong m_3^2 - m_1^2 \cong 4.1\text{x}10^{-3}\ (\text{eV})^2$ for atmospheric neutrino oscillations and with $m_2^2 - m_1^2 \cong 1.4\ \text{x}10^{-5}\ (\text{eV})^2$ for the large mixing angle (LMA) solution to the solar neutrino oscillation data [*33–36*]. The quark pole masses given by (9) are likewise uniformly consistent with the experimental data [*32*], and the predicted value for the top quark pole mass $m_t = 174.241$ GeV may in fact prove to be accurate to four (or more) significant figures.

CONCLUDING REMARKS

The self-interaction mass operator (9) works in a striking manner to yield accurate mass values for the leptons and quarks. Thus, the operator (9) is an admissible rest-frame Hamiltonian for finite-size fundamental fermions on the probable scale $\ell \sim 10^{-18}$ cm. Hence, the underlying theory that embraces finite-size leptons and quarks may produce (9) directly, while also presumably yielding SM in the formal point-particle approximation with $\ell \to 0$. The actual physical value for the fundamental length constant ℓ can be conjectured to equal the Compton length $2\pi/\overline{m}$ suppressed by a weak interaction factor ε; from the empirical values displayed in (10), it then follows that

$$\ell \cong (2\pi/\overline{m})\varepsilon \cong 0.57\text{x}10^{-18}\ \text{cm} \tag{12}$$

Equivalently, (12) states that the bare fermion mass $\overline{m}$ is a particle of size ℓ with *a priori* weak interaction energy suppression: $\overline{m} \cong 2\pi\varepsilon/\ell$. If indeed (12) is the approximate characteristic size for the lepton and quark structural states, this fundamental length should show up in deviations from the SM predictions for the TeV scattering experiments to be performed presently.

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