Modern Physics Letters A, Vol. 22, No. 4, (2007) pp. 283-288

HEURISTIC DEVELOPMENT OF A DIRAC-GOLDHABER MODEL FOR LEPTON AND QUARK STRUCTURE

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All three charged lepton pole masses are given to $O(10^{-5})$ accuracy by $(m_e, m_\mu, m_\tau) = (313.85773 \text{ MeV}) \left[1+ \sqrt{2} \cos\theta_k\right]^2$, where $\theta_k = 2\pi k/3 + 2/9$ with the generation number k = 1, 2, 3. In the context of a Dirac model, it is shown that this empirical formula for the charged leptons admits a satisfactory S_3-parametrized extension for the scale-independent pole masses of quarks and Majorana neutrinos; in particular, the top quark mass emerges as 177.698 GeV. A physical-geometrical interpretation of the mass formula supports Dirac's and Goldhaber's proposals: leptons and quarks of the same generation are identical in size and shape. Specifically, their self-interaction mass appears to be concentrated on spherical surfaces of radii $r_k = \ell\,\mathrm{Re}\,(1+ \sqrt{2}\, z_k)$, in which the Kähler radial complex coordinate z_k is a root of the Calabi-Yau condition $z^3 = \exp(\dot{2}\,i/3)$ for the envelopment of the particle surfaces and ℓ is a fundamental length constant.

Keywords: Lepton and quark masses and structure, Dirac-Goldhaber

1. Introduction

The S_3-symmetric Koide formula[1–7] that relates the charged lepton pole masses

$$m_e + m_\mu + m_\tau = \tfrac{2}{3}\left((m_e)^{1/2} + (m_\mu)^{1/2} + (m_\tau)^{1/2}\right)^2 \tag{1}$$

is accurate to O(10^{-5}), within the data spread for the m_τ with the current experimental values $m_e = 0.5109989\ (1\pm 8.6\text{x}10^{-8})$, $m_\mu = 105.65837\ (1\pm 8.9\text{x}10^{-8})$, and $m_\tau = 1776.99(1\pm 1.55\text{x}10^{-4})$ in MeV. By putting $(m_e\, , m_\mu\, , m_\tau) \equiv (m_{1;\,1}\, , m_{1;\,2}\, , m_{1;\,3})$, (1) admits the S_3-symmetric solution for all three charged lepton masses

$$(m_{1;\,k})^{1/2} = (\overline{m})^{1/2} \left[\ 1+ \sqrt{2}\ \cos\left(2\pi k/3 + 2/9\right)\right] \tag{2}$$

in which the S_3 index, *i.e.*, the *generation number*, appears as k= 1, 2, 3 and $\overline{m}$ is the fundamental mass constant

$$\overline{m} = 313.85773 \text{ MeV} \tag{3}$$

One verifies that (2) satisfies (1) by substituting (2) directly into both sides of (1) and summing over k. Then, by setting $\overline{m}$ equal to its physical value (3), one obtains (in units MeV).

$$\begin{aligned} m_{1;\,1} &= 0.51099650 = m_e\ (1 - 4.70\text{x}10^{-6}) \\ m_{1;\,2} &= 105.65891 = m_\mu\ (1+5.09\text{x}10^{-6}) \\ m_{1;\,3} &= 1776.9764 = m_\tau\ (1-7.63\text{x}10^{-6}) \end{aligned} \tag{4}$$

in which $m_e\, , m_\mu\, , m_\tau$ are the experimental pole masses. The Koide O(10^{-5}) accuracy in (1) reappears in (4), but here for all three charged lepton masses with (3) as the empirical input. Expression (2) had been found independently by a different method in the context of a resurrected preon model.[5]

What is particularly interesting about the mass formula (2) is that it admits a physical-geometrical interpretation along the lines of a Dirac-Goldhaber model[8–10], as described in section **4** below. Here we note that the S_3-symmetry in (2) is subsumed by the association ξ_k = Re $(1+ \sqrt{2}\, z_k)$, where ξ_k is the square-bracketed quantity in (2) and the dimensionless complex number z_k is a root of the cubic equation $z^3 = \exp(2i/3)$. To interpret this physically, we first introduce a model-theoretic formalism that can accommodate lepton-quark structural states.

2. Modernized Dirac model

We assume that leptons and quarks are of finite size (smaller than ~ (1 TeV)$^{-1}$) and have scale-independent pole masses[11,12] that are the eigenvalues of a positive-definite self-adjoint operator **m**, as originally proposed many years ago by Dirac.[8] Following Dirac's approach, we do not address fermion spin, quark color, or the interaction-based mixing of mass eigenstates, the running of the masses, nor the interaction-based transmutation of fermions. The latter are properties of leptons and quarks which must be described, along with their quantum statistical relativistic motion, by anticommuting fields in the established gauge-theoretic Lagrangian manner. Here we concentrate on pole mass and evoke the modern understanding that **m** must derive from a field theory successor to the standard model, and thus we do not make *a priori* restrictive assumptions regarding the form of **m**.

In such a modernized Dirac model, the pole mass eigenvalue equation takes the form

$$\mathbf{m}\,|\,Q;k\rangle = m_{|Q|;\,k}\,|\,Q;k\rangle \tag{5}$$

where the structural fermion states $|\,Q;k\rangle$ are labeled by their charge numbers $Q = (0, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm1)$ and their S_3–symmetry generation numbers k = (1, 2, 3). Since Q = 0 for both neutrinos and antineutrinos, the latter particles are identical for each k and have the structural states $|\,0;k\rangle$; ironically or not, *neutrinos are Majorana in the modernized Dirac model*, with an additional conserved lepton number (associated with interaction symmetry) not appearing on the level of the structure theory. The **m** in (5) is positive-definite self-adjoint, and its positive pole mass eigenvalues are $m_{|Q|;\,k}$. Therefore, it follows that (5) implies the equivalent equation

$[\,(\mathbf{m})^{1/2} - (m_{|Q|;\,k})^{1/2}]\ |\,Q;k\rangle = 0$, in which the uniquely specified square-root operator is positive-definite self-adjoint by definition; thus, the positive square-roots of the pole masses in (1) and (2) are a natural concomitant of (5). Finally, the mass operator **m** is assumed to be expressible as a simple algebraic function of the self-adjoint operators **Q** and **k** with [**Q**, **k**] = 0. Exclusively focused on fermion pole mass, the modernized Dirac model must relate to, and eventually be deduced from, a fundamental field theory. Conversely, an empirical determination of **m**, or equivalently $(\mathbf{m})^{1/2}$, may serve as a heuristic directive for establishing the underlying Langrangian field theory. Implied by the algebraic form of its square-rooted eigenvalues shown in (4) and in (6) below, the S_3-parametrized mass operator **m** is presumably associated with a field theory that brings in the operators **Q**, **k**, the prefactor mass $\overline{m}$ in (2) and the small dimensionless constant ε in (6).

3. Extension of the S_3-parametrized solution to quarks and Majorana neutrinos

Analysis of the quark pole mass data[13] and earlier empirical studies[14–16] yield the extension of (2) as

$$(m_{|Q|;\,k})^{1/2} = (m_{1;\,k})^{1/2}\,[\,|Q| + (9Q^2 + \varepsilon^2)(1 - |Q|)(1 - \hat{k} + 2\hat{k}^2)] \qquad (6)$$

in which $(m_{1;\,k})^{1/2}$ is given by the right side of (2), the quantity

$$\hat{k} \equiv k + 3(1-|Q|)(3|Q|-1) - 2 \;=\; \begin{cases} k-1 & \text{for } |Q| = \tfrac{2}{3} \\ k-2 & \text{for } |Q| = \tfrac{1}{3} \text{ and } |Q| = 1 \\ k-5 & \text{for } |Q| = 0 \end{cases} \qquad (7)$$

and $\varepsilon \cong 6.58\text{x}10^{-4}$ is a small dimensionless constant that effects neutrino mass. The generation number k in (2) is intrinsically S_3-cyclical, for k acts modulo 3 outside its range; hence, with $k \equiv k(\text{mod } 3) = 1, 2, 3$ understood in (7) , the S_3-cyclicality is maintained in (6) via the quadratic term in $\hat{k}$, *viz*. $(1 - \hat{k} + 2\hat{k}^2)$. For the charged leptons with $|Q| = 1$, (6) simply reduces to an identity. For the quarks with $|Q| = \frac{2}{3}$ and $|Q| = \frac{1}{3}$, (6) and (7) yield respectively

$$(m_{2/3;\,k})^{1/2} = (m_{1;\,k})^{1/2} \left[\tfrac{2}{3} + (4+ \varepsilon^2)(\tfrac{1}{3})(4 - 5k + 2k^2)\right]$$

$$(m_{1/3;\,k})^{1/2} = (m_{1;\,k})^{1/2} \left[\tfrac{1}{3} + (1+ \varepsilon^2)(\tfrac{2}{3})(11 - 9k + 2k^2)\right] \qquad (8)$$

from which we obtain (in units MeV)

$$m_u = m_{2/3;\,1} = m_{1;\,1} \left[2+ (\varepsilon^2/3)\right]^2 = 2.04399$$

$$m_c = m_{2/3;\,2} = m_{1;\,2} \left[(10/3) + (2\varepsilon^2/3)\right]^2 = 1173.99$$

$$m_t = m_{2/3;\,3} = m_{1;\,3} \left[10 + (7\varepsilon^2/3)\right]^2 = 177{,}698$$

$$m_d = m_{1/3;\,1} = m_{1;\,1} \left[3 + (8\varepsilon^2/3)\right]^2 = 4.59897$$

$$m_s = m_{1/3;\,2} = m_{1;\,2} \left[1+ (2\varepsilon^2/3)\right]^2 = 105.659$$

$$m_b = m_{1/3;\,3} = m_{1;\,3} \left[(5/3) + (4\varepsilon^2/3)\right]^2 = 4936.05 \qquad (9)$$

with the charged lepton values (4) being employed.

While they are in excellent overall agreement with the experimental data[13], it is of course uncertain whether the quark pole mass values (9) will eventually be confirmed to have the Koide $O(10^{-5})$ accuracy featured by the charged lepton values in (4). The pole masses[11,12] of the u, c, d, s and b quarks are given approximately by their respective "current" masses at 2 GeV, while the pole mass of the top quark is the directly measured experimental mass. Thus, the top quark pole mass value, predicted here as 177.698 GeV, may admit an experimental comparison with $O(10^{-3})$ accuracy in the near future.

Finally, by setting Q = 0 in (6) we obtain the rooted Majorana neutrino mass eigenvalues

$$(m_{0;\,k})^{1/2} = (m_{1;\,k})^{1/2}\, \varepsilon^2 \big(6 - k + 2(k-5)^2\big) \tag{10}$$

which with (4) and $\varepsilon^2 \cong 4.33\text{x}10^{-7}$ imply

$$\begin{aligned} m_1 &\equiv m_{0;\,1} = m_{1;\,1}\, \varepsilon^4\, (37)^2 \cong 1.31\text{x}10^{-4}\ \text{eV} \\ m_2 &\equiv m_{0;\,2} = m_{1;\,2}\, \varepsilon^4\, (22)^2 \cong 9.59\text{x}10^{-3}\ \text{eV} \\ m_3 &\equiv m_{0;\,3} = m_{1;\,3}\, \varepsilon^4\, (11)^2 \cong 4.03\text{x}10^{-2}\ \text{eV} \end{aligned} \tag{11}$$

It follows from (11) that $(m_2)^2 - (m_1)^2 \cong 9.20\text{x}10^{-5}\ (\text{eV})^2$ and $m_3{}^2 - m_2{}^2 \cong 1.53\text{x}10^{-3}\ (\text{eV})^2$, in approximate agreement with neutrino-oscillation data.[17,18] The Majorana character of neutrinos may be confirmed in the near future[19], and refined measurements on neutrino oscillations may approach the mass values (11).

4. Physical-geometrical interpretation of the lepton-quark pole mass formula

From (2) and (6) we obtain

$$m_{|Q|;\,k} = \overline{m}\, \xi_k{}^2\, \chi_k(|Q|) \tag{12}$$

in which

$$\xi_k \equiv 1 + \sqrt{2}\, \cos\,(2\pi k/3 + 2/9) \tag{13}$$

$$\chi_k(|Q|) \equiv \big[\,|Q| + (9Q^2 + \varepsilon^2)(1 - |Q|)(1 - \hat{k} + 2\hat{k}^2)\big]^2 \tag{14}$$

There is an obvious physical-geometrical interpretation that can be given to (12), along the lines of Dirac's and Goldhaber's proposals[8–10] for the structure of finite-size leptons and quarks in their rest frames. Specifically, (12) suggests that the pole masses of elementary fermions are concentrated on spherical surfaces with radii $r_k = \xi_k \ell$ that characterize the leptons and quarks in each generation, where ℓ is a fundamental length construct. Identical in size and shape within a generation, the spherical surfaces have the area $4\pi \ell^2 \xi^2_k$ with $\xi_1 = .04035$ for the first (k = 1), $\xi_2 = .58021$ for the second (k = 2), and $\xi_3 = 2.3794$ for the third generation (k = 3), according to (13). If the Euclidean space is set in the framework of a complex Kähler manifold[20,21] with the dimensionless complex radial coordinate z , (13) is given by the geometrical association $\xi_k = \mathrm{Re}\,(1+ \sqrt{2}\, z_k)$ with z_k a root of the Calabi-Yau condition[20,21] $z^3 = \exp(2i/3)$ for the Kählerian envelopment of the particle surfaces.

Within a generation the pole masses vary with $|Q|$ according to (14), the dimensionless *self-interaction energy density*[8–10] over the spherical surface, here normalized with $\chi_k(1) \equiv 1$. In addition to the electromagnetic self-interaction energy[8], (14) also embodies the strong self-interaction through its baryon number surrogate $3\,|Q|(1-|Q|) = 2\,|B|$ and the weak self-interaction through ε, as proposed by Goldhaber.[9,10]

This physical-geometrical interpretation suggests a geometrically-related tentative value for ℓ, with the prefactor mass constant $\overline{m}$ in (12) presumably proportional to ℓ^{-1}. On the basis of the self-interaction energy interpretation and the magnitude of $\overline{m}$ in (3), we can conjecture that $\overline{m} = \varepsilon/(2\pi \ell)$, which implies that $\ell = \varepsilon/(2\pi\overline{m}) \cong (3.00\ \mathrm{TeV})^{-1}$. The lepton-quark spherical shapes and sizes made probable by (12) may be confirmed by TeV scattering experiments in the future.

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