Research Notes

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Method for the Removal of Free Electrons in a Plasma

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A METHOD for the reduction of the free-electron concentration in a gas plasma is reported in this note. The method is based on the effective "absorption" of free electrons by uniformly distributed "dust" (micron-size) particles of a chemically inert refractory material. It is assumed here that thermionic emission from the dust particles is negligible, an assumption which generally requires the refractory material to behave as a dielectric at the plasma temperature. Essentially a consequence of the relatively large mean velocity of free electrons, a surprisingly small concentration of refractory dust captures almost all electrons in the state of dust–plasma equilibrium.[1] The theoretical details follow.

A plasma electron which enters a micron-size dust particle ordinarily loses its kinetic energy and is captured by the particle. For an idealized spherical dust particle of radius a and electric charge $-Ne$ (due to N captured electrons) the absorption cross section for a free electron of mass m_e and velocity v is

$$\sigma_e = \begin{cases} \pi a^2\left(1 - \dfrac{2Ne^2}{am_e v^2}\right) & \left(\text{for } \dfrac{m_e v^2}{2} \geq \dfrac{Ne^2}{a}\right) \\ 0 & \left(\text{for } \dfrac{m_e v^2}{2} \leq \dfrac{Ne^2}{a}\right). \end{cases}$$

Assuming a Maxwellian distribution for the velocity of electrons, the rate at which free electrons are absorbed by the dust particle is

$$f_e = \int_0^\infty v\sigma_e\left\{n_e\left(\frac{m_e}{2\pi kT}\right)^{\frac{3}{2}}\left[\exp\left(-\frac{m_e v^2}{2kT}\right)\right]4\pi v^2\, dv\right\} = 2a^2 n_e\left(\frac{2\pi kT}{m_e}\right)^{\frac{1}{2}}\left[\exp\left(-\frac{Ne^2}{akT}\right)\right],$$

where n_e is the concentration of free electrons.

In order to operate with simple equations, consider a plasma which contains only one chemical species of positive ions, each ion with the electrical charge $+e$. The dust-particle collision cross section for a singly charged positive ion is

$$\sigma_i = \pi a^2[1 + (2Ne^2/am_i v^2)],$$

where m_i and v are the mass and velocity of the ion, respectively. Assuming a Maxwellian distribution for the velocity of positive ions, the rate at which positive ions collide with the dust particle is

$$f_i = \int_0^\infty v\sigma_i\left\{n_i\left(\frac{m_i}{2\pi kT}\right)^{\frac{3}{2}}\left[\exp\left(-\frac{m_i v^2}{2kT}\right)\right]4\pi v^2\, dv\right\} = 2a^2 n_i\left(\frac{2\pi kT}{m_i}\right)^{\frac{1}{2}}\left(1 + \frac{Ne^2}{akT}\right),$$

in which n_i denotes the concentration of positive ions.

Now the dust particle ordinarily loses an electron during each collision with a positive ion. Hence, $f_e = f_i$ in the state of dust–plasma equilibrium, and it follows that

$$\frac{n_e}{n_i} = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}}\left(1 + \frac{Ne^2}{akT}\right)\left[\exp\left(\frac{Ne^2}{akT}\right)\right].$$

The last equation is valid approximately for an ensemble of dust particles if N refers to the average number of electrons per particle and a refers to the average radius of the particles. Letting n_p denote the concentration of dust particles, the total number of electrons per unit volume is

$$n_e + Nn_p = n_i.$$

Thus, one obtains a transcendental equation for the ratio of free electrons to positive ions n_e/n_i in the state of dust–plasma equilibrium,

$$\frac{n_e}{n_i} = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}}\left[1 + \lambda\left(1 - \frac{n_e}{n_i}\right)\right]\left\{\exp\left[\lambda\left(1 - \frac{n_e}{n_i}\right)\right]\right\},$$

with the parameter $\lambda \equiv e^2 n_i/akTn_p$. The equation shows that n_e/n_i decreases monotonically with decreases in λ. Since $(m_e/m_i)^{\frac{1}{2}}$ is much less than unity, n_e/n_i is much less than unity if λ is of the order of unity. For λ less than or of the order unity, the equation for n_e/n_i reduces to

$$\frac{n_e}{n_i} \cong \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}}(1 + \lambda)(\exp \lambda) \qquad (\lambda \lesssim 1).$$

It should be remarked that the absorption of plasma electrons by dust particles affects the chemical equilibrium of the plasma, in general tending to increase the *total* concentration of electrons (the quantity $n_e + Nn_p = n_i$) in accordance with the principle of Le Chatelier. In particular cases for which the ionization is governed by a one species reaction of the form $M \rightleftarrows M^+ + e^-$, the Saha equation for the reaction predicts the increase in the total concentration of electrons associated with a decrease in the concentration of free electrons. For a chemical species which is highly ionized (n_i large compared to the concentration of neutral particles), the introduction of dust particles produces a small percentage increase in the total electron concentration, and for such cases the total electron concentration is practically insensitive to the absorption of free electrons by the dust particles. On the other hand, for a chemical species which is moderately or only slightly ionized, it is necessary to treat simultaneously the relation which is derived above for n_e/n_i and the condition for chemical equilibrium in order to determine the state of dust–plasma equilibrium.

Consider the result above for typical magnitudes associated with a shock tube plasma: $T = 3000°\text{K}$ and a highly ionized chemical species with $n_i = 10^8$ positive ions/cm^3, $(m_e/m_i)^{\frac{1}{2}} = 5 \times 10^{-3}$. Let a refractory dust with particles of an average radius $a = 10^{-4}$ cm be dispersed uniformly through the plasma. If the specific gravity of the refractory solid equals 5, a dust distribution of 10 mg/liter gives rise to a particle concentration $n_p = (3/2\pi) \times 10^6$ dust particles/cm^3. In this case one finds that $\lambda = 1.17$ and $n_e/n_i = 0.035$, so a small amount of dust absorbs 96.5% of the plasma electrons.

[1] A well-known phenomenon is the establishment of a potential difference between an electrically isolated probe or wall in contact with a plasma [for example, see L. B. Loeb, *Basic Processes of Gaseous Electronics* (University of California Press, 1955), pp. 336–338]. The latter effect results essentially from the large mean velocity of free electrons compared to the mean velocity of positive ions. The establishment of such a negative potential and the refractory dust absorption of free electrons are closely related phenomena.

Magneto-Gas-Dynamic Shock Waves in a Gas with Variable Conductivity

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The methods of deriving the transport properties across a shock have been given in earlier papers by Friedrichs and Kranzer[1] and Kanwal.[2] The basic result for the time rate of change following a particle, through a volume V traversed by the shockwave front $\Sigma(t)$, of a scalar or vector quantity with volume density f is

$$\lim_{V\to 0} \frac{D}{Dt}\int_V f\,dV = \int_\Sigma [fv_n]\,dS. \qquad (1)$$

Here $\mathbf{v}$ is the particle velocity, $\mathbf{n}$ is the unit vector normal to the shock front taken in the sense from ahead of to behind the wave, the brackets [ ] denote the change in the value of the enclosed quantity across the shockwave and the limit on the left-hand side is to be taken in such a way that the volume V becomes simply part of the shock wave. Since the velocity of the shock wave $\mathbf{W} = \mathbf{n}(d\Sigma/dt)$ does not enter (1) it follows that in a discussion of transport properties across a shock wave, the latter may be treated as stationary.

In a gas of finite electrical conductivity σ in the absence of viscous effects and heat conductivity, the (nonrelativistic) equations governing unsteady motion in the presence of a magnetic field are well established. We shall use rationalized mks Giorgi units for the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$. These are related to the current density by Maxwell's equations which, in the absence of free charges or magnetic poles, and if displacement currents are neglected, may be written

$$\nabla\cdot\mathbf{E} = 0, \quad \nabla\cdot\mathbf{H} = 0, \quad \nabla\times\mathbf{H} = \mathbf{j},$$
$$\nabla\times\mathbf{E} = -\mu\,\partial\mathbf{H}/\partial t, \qquad (2\text{a, b, c, d})$$

where μ is the permeability, supposed constant; they are linked to the particle velocity $\mathbf{v}$ of the gas relative to the system of axes in which $\mathbf{E}$ is measured by Ohm's law

$$\mathbf{j} = \nabla\times\mathbf{H} = \sigma(\mathbf{E} + \mu\mathbf{v}\times\mathbf{H}). \qquad (3)$$

From (2b) we derive immediately, by using Gauss' theorem,

$$[H_n] = 0, \qquad (4)$$

and an application of (1) to the integral expressing conservation of mass in the volume V leads at once to

$$[m] = [\rho v_n] = 0, \qquad (5)$$

where m is the mass flux per unit area of the shock front. The law of conservation of momentum gives the equation

$$[m\mathbf{v} + (p + \tfrac{1}{2}\mu H^2)\mathbf{n} - \mu H_n\mathbf{H}] = 0, \qquad (6)$$

where p is the scalar pressure. The equations (4, 5, 6) are in the standard forms and are in no way influenced by the form of Ohm's law ($\sigma \neq 0$).