Dirac model extension for finite-size leptons and quarks in (10+1)D spacetime: Quantum conditions that imply α^{-1}=137.0360824

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Abstract. – In a Dirac model extension to 10 spatial dimensions (3 external plus 7 internal), leptons and quarks at rest are viewed to have a universal size and prolate spheroid shape, with mass and charge differences wholly attributed to their energy and charge densities through the universal spheroid. It is noted that simple volumetric quantum conditions on the 10D prolate spheroid and on its 4D prolate spheroid projection imply the fine-structure constant theoretical value $\alpha = (137.0360824)^{-1}$.

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Half a century ago, Dirac [1] proposed a finite-size spherical model for an electron at rest, a model which derived in part from the conjectured existence of a fundamental length ℓ. The Dirac model has been revisited with elaborations more recently [2–5], subject to the contemporary awareness that ℓ must be smaller than about $1\times10^{-18}\,\text{cm},$ a size compatible with the point-particle approximation of QED and the standard model for leptons and quarks with an indication of finite-size still below current experimental resolution. Like crystals forming in a fluid medium, leptons and quarks are viewed as structured entities within their respective (motion and interaction) controlling quantum fields.

Here we consider a Dirac model extension in string-theory inspired (10+1)D spacetime. Without conjecturing the magnitude of ℓ, we choose the fundamental length unit to be ℓ, and employ physical units that make $\ell = 1$, along with $\hbar = 1$ and $c = 1$. We investigate the most natural universal shape of leptons and quarks in 10 spatial dimensions for particles that are spherical at rest in the 3D external space cross-section. By requiring symmetry between the external (x_1, x_2, x_3) space and the internal (x_4, x_5, x_6) and (x_7, x_8, x_9) spaces, we are led to consider the 10D prolate spheroid

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$$R_{10} \equiv \left\{ \; x_1, \ldots, x_9, x_{10} : \left(a^{-2} \sum_{i=1}^{9} x_i^2 + b^{-2} x_{10}^2 \right) \leqq 1 \; \right\} \tag{1}$$

with the volume

$$V_{10} = \int_{R_{10}} \left(\prod_{i=1}^{9} dx_i \right) dx_{10} = \frac{\pi^5}{5!} a^9 b \quad . \tag{2}$$

In (1) and (2), *a* is the semi-minor axis, as well as the radius of the lepton or quark in the external (x_1, x_2, x_3) space, while *b* is the semi-major axis in the internal x_{10} direction, the axis of revolution of the prolate spheroid, possibly associated with lepton-quark spin. Leptons and quarks at rest are viewed to have the universal shape given by (1), with their physical size deferred to future experimental determination of ℓ, and with their mass and charge differences attributed to different energy and charge densities through their R_{10} volume.

Next, let us associate local electromagnetic gauge transformations with x_{10} and *b*, as follows. Wavefunctions of charged particles are modified by a phase factor under local electromagnetic gauge transformations:

$$\psi \xrightarrow{\gamma} (\exp(i\varepsilon\gamma)) \; \psi \tag{3}$$

where ε denotes the particle's charge and γ is the gauge parameter. Since the basic unit of electric charge is $\varepsilon = e/3$ and all lepton and quark charges are positive or negative multiples thereof, all gauge transformations are contained in the range $-b \leqq \gamma \leqq b$ with

$$b \equiv 3\pi / e \quad . \tag{4}$$

We identify the internal coordinate x_{10} to in fact be γ, $x_{10} \equiv \gamma$, and concomitantly assign *b* to have the physical value (4) in (1) and (2).

Now let $P = P^2$ denote the projection operator that for $P = 0$ maps the 10D prolate spheroid (1) into the 4D prolate spheroid

$$R_4 \equiv \left\{ \; x_1, x_2, x_3, x_{10} : \left(a^{-2} \sum_{i=1}^{3} x_i^2 + b^{-2} x_{10}^2 \right) \leqq 1 \; \right\} \tag{5}$$

with the volume

$$V_4 = \int_{R_4} dx_1 dx_2 dx_3 dx_{10} = \frac{\pi^2}{2} a^3 b \quad . \tag{6}$$

Note that (1) and (5) are subsumed by the single definition

$$R_{4+6P} \equiv \left\{ \; x_1, x_2, x_3, Px_4, \ldots Px_9, x_{10} : \left(a^{-2} \sum_{i=1}^{3+6P} x_i^2 + b^{-2} x_{10}^2 \right) \leqq 1 \; \right\} \tag{7}$$

Viewing P as a quantum operator with eigenvalues $P = 0$ and 1, we seek the simplest mathematically compatible volumetric quantum conditions for (2) and (6). We soon find

$$V_{4+6P} = \frac{1}{4} a^{2-3P} b^{2+P}, \tag{8}$$

which in combination with (2) and (6) yields

$$\frac{\pi^5}{5!} a^9 b = \frac{1}{4} a^{-1} b^3 \qquad \text{and} \qquad \frac{\pi^2}{2} a^3 b = \frac{1}{4} a^2 b^2 . \tag{9}$$

Solving eqs. (9), we obtain

$$a = (120/\pi)^{1/8} = 1.576717623 \tag{10}$$

$$b = 2\pi^2 a = 31.12315838 \tag{11}$$

From (11) and (4) it follows that

$$e = 3\pi / b = 0.3028220288 \tag{12}$$

Note that the left-side members of (11) and (12) imply

$$(2\pi a)(e/3) = 1, \tag{13}$$

which states that the lepton or quark circumference in the external (x_1, x_2, x_3) space, $2\pi a$, is the reciprocal of the basic charge unit (e/3). Finally, (12) yields the fine-structure constant value

$$\alpha = e^2 / 4\pi = (137.0360824)^{-1}. \tag{14}$$

The latter theoretical value is less than the CODATA experimental value, $\alpha_{\exp} = (137.0359991)^{-1}$, by a mere 4.5×10^{-9}.

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