Conditions on the Higgs-Yukawa couplings for lepton mass

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Abstract

Certain cubic conditions on the Higgs-Yukawa couplings for the three-generation masses of charged-leptons and neutrinos are reported here. Embodying an S_3 symmetry, the associated mass values for e, μ, τ and the three neutrinos are in striking agreement (correspondence accuracy of $O(10^{-6})$ for the e, μ, τ masses) with the experimental mass values, suggesting that the coupling theory features the conditions and the S_3 symmetry.

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1. Introduction

The quantum field theory for fundamental matter is given to high accuracy by the three-generation standard model [1–3] based on the $SU(2)_L \times U(1)_Y$ symmetry group and the Higgs mechanism [4] for electroweak symmetry breaking. The effective phenomenological free lepton part of the Lagrangian emerges as [5]

$$\mathcal{L}_D = \sum_{|Q|=0}^{1} \sum_{k=1}^{3} \overline{\psi}_{|Q|,k} (\gamma^\mu \partial_\mu + m_{|Q|,k}) \psi_{|Q|,k} \tag{1}$$

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where $\psi_{|Q|,k}$ are the four-component complex-valued Dirac fields, $|Q| = 1$ for the three generations (k = 1, 2, 3) of charged-leptons (e, μ, τ), and $|Q| = 0$ for the three generations of unmixed (mass-eigenstate) neutrinos. The lepton masses $m_{|Q|,k}$ are all proportional to the Higgs scalar-field vacuum expectation value, 246 GeV, and individually proportional to their respective Higgs-Yukawa couplings, which are dimensionless and small. With the absorptions of the latter factors assumed tacitly, we refactor the mass values $m_{|Q|,k}$ into mean-mass values $\hat{m}_{|Q|}$ times the norm-squared of dimensionless complex-numbers $\xi_{|Q|,k}$ for $|Q| = 0$ and 1 and $k = 1, 2, 3$:

$$m_{|Q|,k} = \hat{m}_{|Q|} \, |\xi_{|Q|,k}|^2 \tag{2}$$

The purpose of the present communication is to report cubic determining conditions on the complex-numbers $\xi_{|Q|,k}$ in (2). Presumably relating an S_3 symmetry in the Higgs-Yukawa couplings, the resulting values for the $\xi_{|Q|,k}$ yield the six lepton masses in striking agreement with the experimental mass values (a correspondence accuracy of $O(10^{-6})$ for the charged-leptons), with the mean-value masses $\hat{m}_1$ and $\hat{m}_0$ prescribed suitably. Thus, the conditions specify the Higgs-Yukawa couplings to within an overall normalization.

2. Cubic conditions

Empirical analysis and model theoretic studies [6] suggest consideration of the cubic conditions

$$(\xi_{|Q|,k})^3 = (\exp(2i/3)) \; [1 + (1+ |Q|)(\mathrm{Re}\, \xi_{|Q|,k})/\sqrt{2}\, |\xi_{|Q|,k}|]^3 \tag{3}$$

By putting $\xi_{|Q|,k} = \rho \exp(i\theta)$ with ρ and θ real, ρ positive and θ defined modulo 2π, conditions (3) resolve into the Pascal limaçon relation for $\rho = \rho(\theta)$

$$\rho = 1 + (1+ |Q|)(\cos\theta)/\sqrt{2} \qquad (4)$$

and the phase-angle constraint $\exp(3i\theta) = \exp(2i/3)$ which fixes θ to one of three values:

$$\theta = \theta_k \equiv \frac{2\pi}{3}k + \frac{2}{9} \qquad \text{for } k \equiv 1, 2, 3 \qquad (5)$$

Hence, the cubic conditions (3) have the general solution

$$\xi_{|Q|,\,k} = (\exp(i\theta_k))\ [1 + (1+ |Q|)(\cos\theta_k)/\sqrt{2}\,] \qquad (6)$$

in which there appears the S_3 labeling parameter k, the generation index. By virtue of (6), the lepton masses (2) are given by

$$m_{|Q|,\,k} = \hat{m}_{|Q|}\,[1 + (1+ |Q|)(\cos\theta_k)/\sqrt{2}\,]^2 \qquad (7)$$

3. Experimental correspondence

Let the $m_{|Q|}$ be prescribed empirically as

$$\hat{m}_1 \equiv 313.85773 \text{ MeV} \qquad \hat{m}_0 = 1.80 \times 10^{-2} \text{ eV} \qquad (8)$$

Then (7) and (8) yield (in units MeV)

$$m_{1,k} = \hat{m}_1\,(1+ \sqrt{2}\,\cos\theta_k)^2 \quad \Rightarrow \quad \begin{aligned} m_{1,1} &= 0.51099651 = m_e\,(1 - 4.72\times10^{-6}) \\ m_{1,2} &= 105.65891 = m_\mu\,(1+5.10\times10^{-6}) \\ m_{1,3} &= 1776.9765 = m_\tau\,(1-7.61\times10^{-6}) \end{aligned} \qquad (9)$$

where m_e, m_μ , m_τ on the right sides of (9) are the precise experimental values [7] : m_e =

$0.51099892\,(1 \pm 7.8 \times 10^{-8})$, $m_\mu = 105.658369\,(1 \pm 8.5 \times 10^{-8})$, and $m_\tau = 1776.99(1 \pm 1.58 \times 10^{-4})$ in MeV. A striking agreement with the experimental masses is evidenced by (9), with an $O(10^{-6})$ accuracy of correspondence. Moreover, for the neutrinos with $|Q| = 0$ in (7) and (8), one obtains

$$m_{0,k} = \hat{m}_0 \left(1 + \frac{1}{\sqrt{2}} \cos \theta_k\right)^2 \Rightarrow \begin{array}{l} m_{0,1} = 4.87 \times 10^{-3}\ \text{eV} \\ m_{0,2} = 1.12 \times 10^{-2}\ \text{eV} \\ m_{0,3} = 5.14 \times 10^{-2}\ \text{eV} \end{array} \tag{10}$$

The neutrino masses shown in the final members of (10) yield $(m_{0,2})^2 - (m_{0,1})^2 = 1.02 \times 10^{-4}\ (\text{eV})^2$ and $(m_{0,3})^2 - (m_{0,2})^2 = 2.52 \times 10^{-3}\ (\text{eV})^2$, in satisfactory agreement with the median values obtained from neutrino-oscillation data [8,9]. On-going neutrino oscillation experiments with refined measurements will further test the degree of accuracy in the neutrino masses given by the second member of (8) and (10).

4. Concluding remarks

Surrogates for the Higgs-Yukawa couplings, the dimensionless complex-numbers $\xi_{|Q|,k}$ are specified to within an S_3 permutation of the generation index $k = 1, 2, 3$ by the conditions (3) for both $|Q| = 1$ charged-leptons and $|Q| = 0$ neutrinos. Since the norm-squares $|\xi_{|Q|,k}|^2$ are proportional to the corresponding Higgs-Yukawa couplings, the latter are also specified by (3) to within an overall normalization factor. Thus, the Higgs-Yukawa couplings can be expected to satisfy conditions associated with (3) and the S_3 symmetry shown by (5) and (7).

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